ECL 4340

POWER SYSTEMS

LECTURE 15

SYMMETRICAL FAULTS, GENERATOR MODEL

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ANNOUNCEMENTS

- Be reading Chapter 7.
- HW 8 is uploaded; Due date: October 28.
- Exam II is on November 8, Tuesday.

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Fault Analysis

- The cause of electric power system faults is insulation breakdown
- This breakdown can be due to a variety of different factors
  - lightning
  - wires blowing together in the wind
  - animals or plants coming in contact with the wires
  - salt spray or pollution on insulators

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Fault Types

- There are two main types of faults
  - symmetric faults: system remains balanced; these faults are relatively rare but are the easiest to analyze so we'll consider them first.
  - unsymmetric faults: system is no longer balanced; very common, but more difficult to analyze
- Most common type of fault on a three-phase system by far is the single line-to-ground (SLG), followed by the line-to-line faults (LL), double line-to-ground (DLG) faults, and balanced three phase faults
  - On very high voltage lines faults are practically always single line-to-ground due to large conductor spacing

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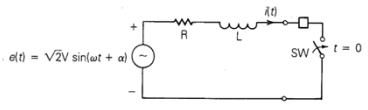
RL Circuit Analysis

1. Circuit Transients



$$L\frac{di(t)}{dt} + Ri(t) = \sqrt{2}V\sin(\omega t + \alpha) \quad t \geq 0$$

The solution to (7.1.1) is

$i(t) = i_{ac}(t) + i_{dc}(t)$; asymmetrical fault current

$$= \frac{\sqrt{2}V}{Z}[\sin(\omega t + \alpha - \theta) - \sin(\alpha - \theta)e^{-t/T}] \text{ A}$$

symmetrical, steady-state dc offset

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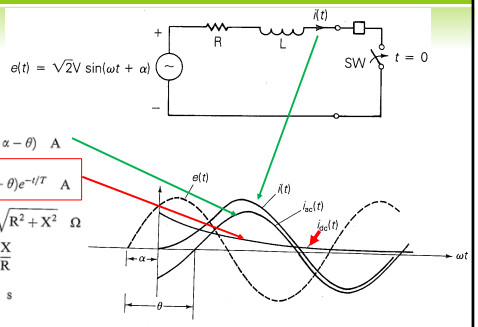
RL Circuit Analysis



$$i_{ac}(t) = \frac{\sqrt{2}V}{Z}\sin(\omega t + \alpha - \theta) \quad A$$

$$i_{dc}(t) = -\frac{\sqrt{2}V}{Z}\sin(\alpha - \theta)e^{-t/T} \quad A$$

$$Z = \sqrt{R^2 + (\omega L)^2} = \sqrt{R^2 + X^2} \quad \Omega$$

$$\theta = \tan^{-1}\frac{\omega L}{R} = \tan^{-1}\frac{X}{R}$$

$$T = \frac{L}{R} = \frac{X}{\omega R} = \frac{X}{2\pi f R} \quad s$$

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RL Circuit Analysis

$$i_{ac}(t) = \frac{\sqrt{2}V}{Z}\sin(\omega t + \alpha - \theta) \quad A$$

$$i_{dc}(t) = -\frac{\sqrt{2}V}{Z}\sin(\alpha - \theta)e^{-t/T} \quad A$$

Note: at $t=0$, $i(t) = 0$

$\Rightarrow$ steady-state current = dc offset

If $\alpha - \theta = 0, \pi$, dc offset $\equiv 0$

$\alpha - \theta = \mp\frac{\pi}{2}$, dc offset $= \pm\frac{\sqrt{2}V}{Z} = \pm\frac{V_{max}}{Z}$

(a) $\alpha - \theta = 0$

(b) $\alpha - \theta = -\frac{\pi}{2}$: maximum dc offset

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RMS for Fault Current

$$i_{ac}(t) = \frac{\sqrt{2}V}{Z}\sin(\omega t + \alpha - \theta) \quad A$$

$$i_{dc}(t) = -\frac{\sqrt{2}V}{Z}\sin(\alpha - \theta)e^{-t/T} \quad A$$

RMS Values: for $i(t)$, asymmetrical?

$i_{ac}(t) = \frac{\sqrt{2}V}{Z}\sin(\omega t + \alpha - \theta) \Rightarrow I_{ac} = \frac{V}{Z}$, rms

$i_{dc}(t) = -\frac{\sqrt{2}V}{Z}\sin(\alpha - \theta)\,e^{-t/T}$

: maximum at $\alpha - \theta = -\frac{\pi}{2}$

∴ The largest fault current is at $\alpha - \theta = -\frac{\pi}{2}$:

$$i(t) = \sqrt{2}\,I_{ac}\left[\sin\left(\omega t - \frac{\pi}{2}\right) + e^{-t/T}\right]$$

$$I_{rms}(t) = \sqrt{[I_{ac}]^2 + [I_{dc}(t)]^2}$$

$$= \sqrt{[I_{ac}]^2 + [\sqrt{2}I_{ac}e^{-t/T}]^2}$$

$$= I_{ac}\sqrt{1 + 2e^{-2t/T}} \quad A \qquad (7.1.10)$$

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RMS FOR FAULT CURRENT

$$I_{rms}(t) = \sqrt{[I_{ac}]^2 + [I_{dc}(t)]^2}$$
$$= \sqrt{[I_{ac}]^2 + [\sqrt{2}I_{ac}e^{-t/T}]^2}$$
$$= I_{ac}\sqrt{1 + 2e^{-2t/T}} \quad A \qquad (7.1.10)$$

It is convenient to use $T = X/(2\pi f R)$ and $t = \tau/f$, where τ is time in cycles, and write (7.1.10) as

$$I_{rms}(\tau) = K(\tau)I_{ac} \quad A \qquad (7.1.11)$$

$K(\tau)$: asymmetry factor

where

$$K(\tau) = \sqrt{1 + 2e^{-4\pi\tau/(X/R)}} \quad \text{per unit} \qquad (7.1.12)$$

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RMS FOR FAULT CURRENT

DC offset: $I_{dc} = \sqrt{2}\, I_{ac}\, e^{-t/T}$

Time constant: $T = \frac{L}{R} = \frac{2\pi f L}{2\pi f R} = \frac{X}{2\pi f R}$

Time: $t = \frac{\tau}{f}$, τ = cycles

Asymmetry factor:

$K(\tau) = \sqrt{1 + 2e^{-2t/T}}$,

$= \sqrt{1 + 2e^{-4\pi\tau/(X/R)}}$

$= \sqrt{3}$ @ $\tau = 0$

$= 1$ for large τ

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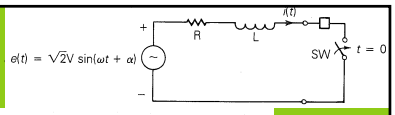

EXAMPLE 7.1 Fault currents: R–L circuit with ac source

A bolted short circuit occurs in the series R–L circuit of Figure 7.1 with V = 20 kV, X = 8 Ω, R = 0.8 Ω, and with maximum dc offset. The circuit breaker opens 3 cycles after fault inception. Determine (a) the rms ac fault current, (b) the rms "momentary" current at $\tau = 0.5$ cycle, which passes through the breaker before it opens, and (c) the rms asymmetrical fault current that the breaker interrupts.

SOLUTION

a. From (7.1.9),

$$I_{ac} = \frac{20 \times 10^3}{\sqrt{(8)^2 + (0.8)^2}} = \frac{20 \times 10^3}{8.040} = 2.488 \quad kA$$

b. From (7.1.11) and (7.1.12) with $(X/R) = 8/(0.8) = 10$ and $\tau = 0.5$ cycle,

$$K(0.5 \text{ cycle}) = \sqrt{1 + 2e^{-4\pi(0.5)/10}} = 1.438$$
$$I_{momentary} = K(0.5 \text{ cycle})I_{ac} = (1.438)(2.488) = 3.576 \quad kA$$

c. From (7.1.11) and (7.1.12) with $(X/R) = 10$ and $\tau = 3$ cycles,

$$K(3 \text{ cycles}) = \sqrt{1 + 2e^{-4\pi(3)/10}} = 1.023$$
$$I_{rms}(3 \text{ cycles}) = (1.023)(2.488) = 2.544 \quad kA \quad \blacksquare$$

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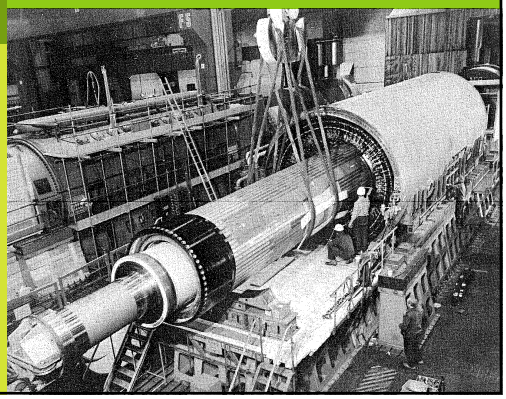
Synchronous Generator



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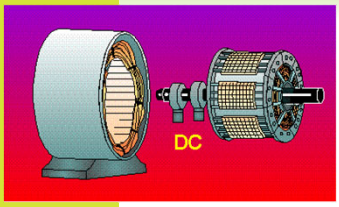
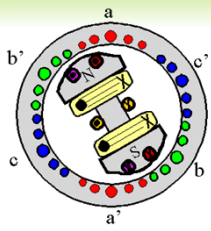
Synchronous Generator





http://energy-conversation.blogspot.com/2010/02/synchronous-generator.html

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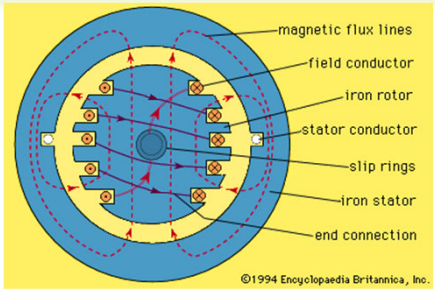
Synchronous Generator



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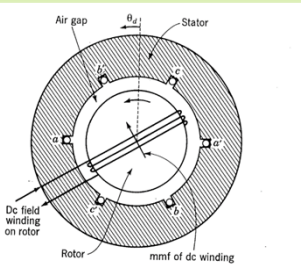
Synchronous Generator



2-pole synchronous machine

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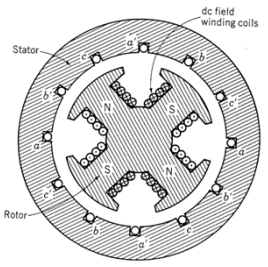
Synchronous Generator



4-pole synchronous machine

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Generator Modeling

Flux linkage on phase-a stator winding:

$\phi = \Phi_f \cos \omega t$

Faraday's law: voltage induced on coil-a:

$e = -N \frac{d\phi}{dt}$

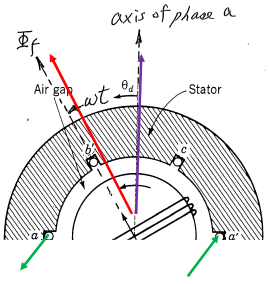
$= \omega N \Phi_f \sin \omega t$

$= \underbrace{\omega N \Phi_f}_{E_m} \cos(\omega t - 90°)$: lags by 90°

Field voltage:

$|E_f| = \frac{E_m}{\sqrt{2}} = \frac{2\pi f N \Phi_f}{\sqrt{2}}$

$= 4.44 f N \Phi_f$



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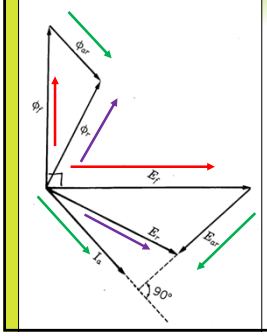
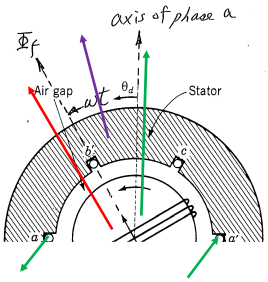
Generator Modeling

Amature Reaction:





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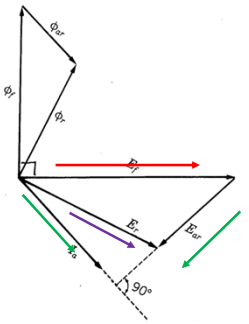
Generator Modeling

Amature Reaction:



Since E_{ar} lage I_a by 90°,

$$E_{ar} = -jI_a X_{ar}$$

$$E_r = E_f + E_{ar} = E_f - jI_a X_{ar}$$

I_a: armature current

Φ_{ar}: armature reaction flux

E_{ar}: armature reaction voltage

Φ_r: resultant flux

E_r: resultant voltage

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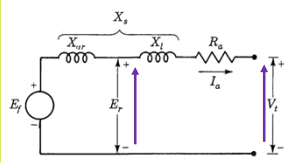
Generator Modeling



Equivalent Circuit

Terminal Voltage:

$$V_t = E_r - jI_a X_l$$

$$V_t = E_f - I_a(R_a + jX_s)$$

$X_s = X_{ar} + X_l$

: synchronous reactance

R_a: armature resistance

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Generator Modeling

- Effect of Armature Current:

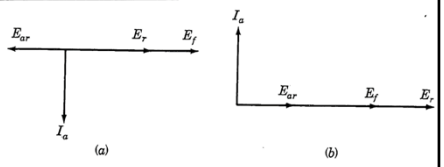

Figure 6.7 Phasor diagrams showing the relation between E_f and E_{ar} when current delivered by a generator is (*a*) lagging E_f by 90° and (*b*) leading E_f by 90°.

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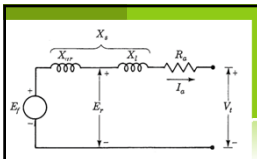
Generator Modeling



Equivalent Circuit

- Effect of Excitation:

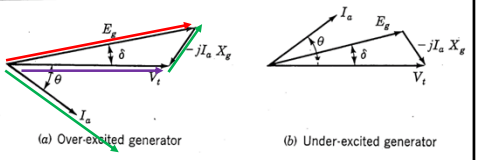


Figure 6.9 Phasor diagrams of (*a*) overexcited and (*b*) underexcited generator. I_a is current delivered by the generator.

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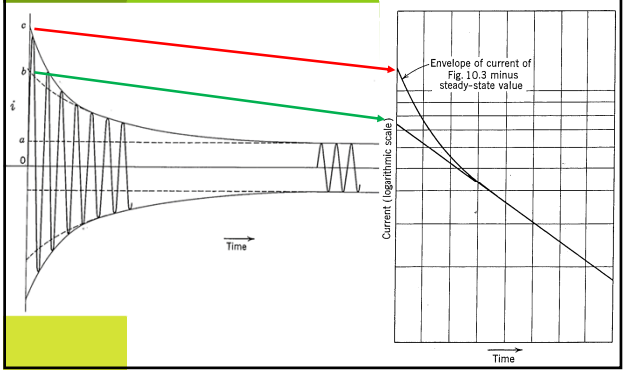
Three-Phase Short Circuit



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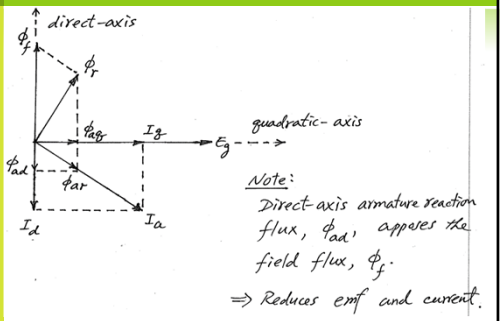
THREE-PHASE SHORT CIRCUIT



Note:
Direct-axis armature reaction flux, ϕ_{ad}, opposes the field flux, ϕ_f.
$\Rightarrow$ Reduces emf and current.

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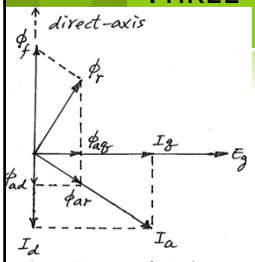
THREE-PHASE SHORT CIRCUIT



During fault, short circuit current is highly reactive due to negligible resistance. This tends to reduce airgap flux : Armature Reaction
$\Rightarrow$ Tends to reduce I_a.

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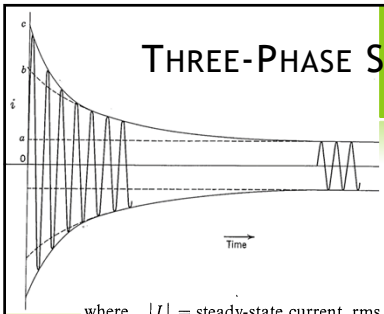
THREE-PHASE SHORT CIRCUIT



$$|I| = \frac{oa}{\sqrt{2}} = \frac{|E_g|}{X_d}$$

$$|I'| = \frac{ob}{\sqrt{2}} = \frac{|E_g|}{X'_d}$$

$$|I''| = \frac{oc}{\sqrt{2}} = \frac{|E_g|}{X''_d}$$

where $|I|$ = steady-state current, rms value
$|I'|$ = transient current, rms value excluding dc component
$|I''|$ = subtransient current, rms value excluding dc component
X_d = direct-axis synchronous reactance
X'_d = direct-axis transient reactance
X''_d = direct-axis subtransient reactance
$|E_g|$ = rms voltage from one terminal to neutral at no load
oa, ob, oc = intercepts shown in Fig. 10.3.

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THREE-PHASE SHORT CIRCUIT

Theorem of Constant Flux Linkages:

Flux linkage cannot change instantaneously. Initially forced to flow through high reluctance path, i.e., low reactance path.

Reluctance: $\mathcal{R}$

Flux: $\phi = \frac{\mathcal{F}}{\mathcal{R}} = \frac{NI}{\mathcal{R}} \triangleq LI$

Then moves towards the lower reluctance path, i.e., high reactance path.

$\Rightarrow$ Time-varying inductance $L(t)$ or reactance $X(t) = \omega L(t)$.

Define: $X_d'' < X_d' < X_d$

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THREE-PHASE SHORT CIRCUIT

Instantaneous fault current:

$$i_{ac}(t) = \sqrt{2}E_g\left[\left(\frac{1}{X_d''} - \frac{1}{X_d'}\right)e^{-t/T_d''} + \left(\frac{1}{X_d'} - \frac{1}{X_d}\right)e^{-t/T_d'} + \frac{1}{X_d}\right]\sin\left(\omega t + \alpha - \frac{\pi}{2}\right)$$

at $t=0$: $I_{ac}(0) = \frac{E_g}{X_d''} = I''$, subtransient current, $\theta = \frac{\pi}{2}$ for $R=0$

T_d'', d-axis subtransient current time constant

for large t, $I_{ac}' = \frac{E_g}{X_d'}$, transient current,

for much larger t, $I_{ac}(\infty) = \frac{E_g}{X_d} = I$, steady-state current

Max dc offset: @ $\alpha = 0$, $i_{dc\,max}(t) = \frac{\sqrt{2}E_g}{X_d''}e^{-t/T_A} = \sqrt{2}I''e^{-t/T_A}$

= peak of ac @ $t=0$.

T_A: armature time constant

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GENERATOR SHORT CIRCUIT EXAMPLE

- A 500 MVA, 20 kV, 3ϕ is operated with an internal voltage of 1.05 pu. Assume a solid 3ϕ fault occurs on the generator's terminal and that the circuit breaker operates after three cycles. Determine the fault current. Assume

$X_d'' = 0.15$, $X_d' = 0.24$, $X_d = 1.1$ (all per unit)

$T_d'' = 0.035$ seconds, $T_d' = 2.0$ seconds

$T_A = 0.2$ seconds

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Generator S.C. Example, cont'd

Substituting in the values

$$I_{ac}(t) = 1.05\left[\frac{1}{1.1}+\left(\frac{1}{0.24}-\frac{1}{1.1}\right)e^{-t/2.0}+\left(\frac{1}{0.15}-\frac{1}{0.24}\right)e^{-t/0.035}\right]$$

$$I_{ac}(0) = 1.05/0.15 = 7 \text{ p.u.}$$

$$I_{base} = \frac{500\times10^6}{\sqrt{3}\,20\times10^3} = 14{,}433 \text{ A} \quad I_{ac}(0) = 101{,}000 \text{ A}$$

$$I_{dc}(0) = 101 \text{ kA}\times\sqrt{2}\,e^{t/0.2} = 143 \text{ k A} \quad I_{rms}(0) = 175 \text{ kA}$$

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Generator S.C. Example, cont'd

Evaluating at t = 0.05 seconds for breaker opening

$$I_{ac}(0.05) = 1.05\left[\frac{1}{1.1}+\left(\frac{1}{0.24}-\frac{1}{1.1}\right)e^{-0.05/2.0}+\left(\frac{1}{0.15}-\frac{1}{0.24}\right)e^{-0.05/0.035}\right]$$

$$I_{ac}(0.05) = 70.8 \text{ kA}$$

$$I_{dc}(0.05) = 143\times e^{-0.05/0.2} \text{ kA} = 111 \text{ k A}$$

$$I_{rms}(0.05) = \sqrt{70.8^2+111^2} = 132 \text{ kA}$$

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